**Lecture 15.**

 **Some curve sketching.**

 **Analysis of function using the derivative, plotting graph of function.**

While constructing the graph of function, find:

1. Domain of function
2. Symmetry, periodicity
3. Points of discontinuity
4. Asymptotes
5. Constancy of sign
6. Extremum of function (monotonicity)
7. Points of inflection (concavity)
8. The graph of function

These elements help to determine the general nature of the graph of the function and to obtain a mathematically correct outline of it.

**Example.** Construct the graph of the function

 

**Solution:**

1. *Domain of function .* The function exists everywhere except at the points $x=\pm 2.$

 

1. *Symmetry, periodicity .* Since

 

we conclude that the function is odd, and therefore the graph of function is symmetric about the origin. This simplifies construction of the graph. *f* is not a periodic function.

1. *Points of discontinuity* are $x=-2$ and $x=2.$

  =-,  =+.

The straight lines $x=\pm 2$ are vertical asymptotes of the graph.

1. *Asymptotes.* We seek inclined asymptotes, and find

  ,

 .

Consequently, the straight line $y=x$ is inclined asymptote.

1. *Constancy of sign.* We are going to find the points of intersection of the curve with the coordinate axes.

Putting $y=0,$ we find  (the point of intersection of the

curve with the axis of abscissas); and putting $x=0,$ we get



(the point of intersection of the curve with the axis of ordinate).

1. *Extremum of function (monotonicity).* We find the critical points, and have

 

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It is convenient to tabulate the results of such an investigation. It will be noted that due to the oddness of the function $y,$ it is enough to calculate for $x\geq 0;$ (the left-hand half of the graph is constructed by the principle of odd symmetry).

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  | 0 |  |  |  |  |
|  | - | 0 | - | - | 0 | + |
|  |  | 0 |  |  |  |  |
|  | decreases | there is no extrema | decreases | decree-ses | min | increases |

1. *Points of inflection (concavity).* We have

 . , at points  the function is not defined.

We form a table, including the characteristic points.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |   | 0 |  |  |
|  | + | 0 | - | + |
|  |  | 0 |  |  |
|  | Concave up | (0;0) is a point of inflection. | Concave down | Concave up |

1. *The graph of function*. Using the results of the investigation, we construct the graph of the function.

